Let X and Y be i.i.d U(0,1)

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$$f_X(x) = 1 \quad ; \quad 0 < x < 1$$

$$f_Y(y) = 1 \quad ; \quad 0 < y < 1$$

Since X and Y are independent

So,

$$f_{X,Y}(x,y) = f_X(x) \times f_Y(y)$$

$$f_{X,Y}(x,y) = 1 \times 1$$

$$f_{X,Y}(x,y) = 1 \quad ; \quad \begin{array}{l} 0 < x < 1 \\ 0 < y < 1 \end{array}$$

Now $\quad Z = \dfrac{Y}{X}$

Suppose,

$$U = X$$

$$\Rightarrow Z = \frac{Y}{U}$$

$$\Rightarrow Y = ZU$$

$$|J| = \begin{vmatrix} \frac{\partial x}{\partial u} & \frac{\partial x}{\partial z} \\ \frac{\partial y}{\partial u} & \frac{\partial y}{\partial z} \end{vmatrix}$$

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$$|J| = \begin{vmatrix} 1 & 0 \\ z & u \end{vmatrix} = u$$

$$f_{U,Z}(u,z) = 1 \times u = u$$

Now we shall get the range of u and z

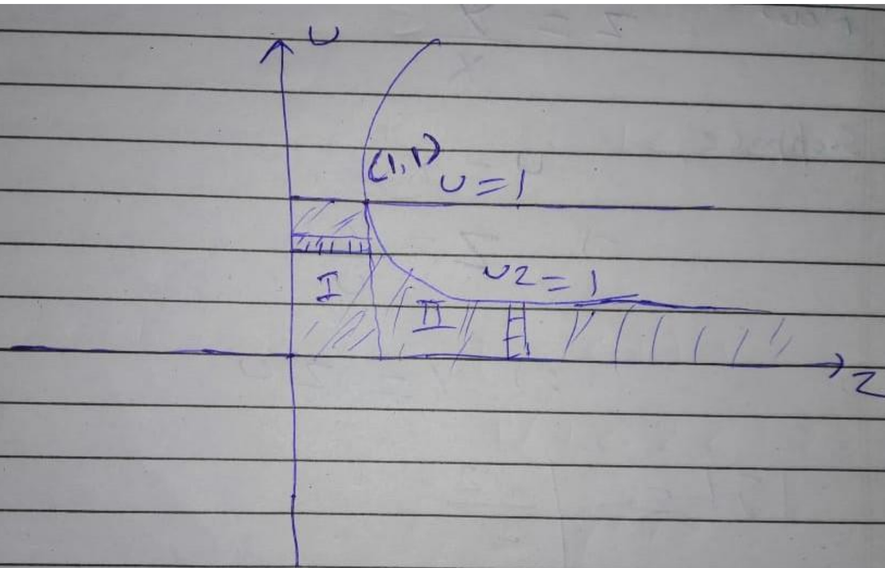
$\because \quad 0 < x < 1$

$0 < u < 1$

$0 < y < 1$

$0 < uz < 1$

$uz = 0, \quad uz = 1$



For region I

$0 < u < 1$

$0 < z < 1$

For region II

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$$0 < u < \frac{1}{2}$$

$$1 < z < \infty$$

Thus, joint PDF of U and Z can be written as —

$$f_{U,Z}(u,z) = \begin{cases} u \; ; & 0 < u < 1 \\ & 0 < z < 1 \\ u \; ; & 0 < u < \frac{1}{2} \\ & 1 < z < \infty \end{cases}$$

Marginal PDF of Z can be obtained as —

$$f_Z(z) = \int_0^1 f_{U,Z}(u,z)\,du$$

$$f_2(z) = \begin{cases} \displaystyle\int_0^1 u\,du & ; \ 0 < z < 1 \\[2ex] \displaystyle\int_0^{1/z} u\,du & ; \ 1 < z < \infty \end{cases}$$

$$f_2(z) = \begin{cases} \left[\dfrac{u^2}{2}\right]_0^1 & ; \ 0 < z < 1 \\[2ex] \left[\dfrac{u^2}{2}\right]_0^{1/z} & ; \ 1 < z < \infty \end{cases}$$

$$f_Z(z) = \begin{cases} \frac{1}{2} & ; \; 0 < z < 1 \\ \frac{1}{2z^2} & ; \; 1 < z < \infty . \end{cases}$$

$$P(Z \le 1.25) = P(Z \le 1) + P(1 < Z < 1.25)$$

$$= \int_0^1 f_Z(z)\,dz + \int_1^{1.25} f_Z(z)\,dz$$

$$= \int_0^1 \frac{1}{2}\,dz + \int_1^{1.25} \frac{1}{2z^2}\,dz$$

$$= \frac{1}{2}\left[z\right]_0^1 + \frac{1}{2}(-2+1)\left[\frac{1}{z}\right]_1^{1.25}$$

$$= \frac{1}{2} + \left(\frac{-1}{2}\right)\left[\frac{1}{1.25} - 1\right]$$

$$= \frac{1}{2} + \frac{1}{2} \times \frac{0.25}{1.25} = \frac{1}{2} + \frac{1}{10}$$

$$= 0.6$$

$$\boxed{P(Z \leq 1.25) = 0.6}$$